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Simplified procedure for vibration analysis and dynamic balancing in mechanical systems with beats frequency

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ABSTRACT

Beats frequency due to the modulation of vibration can occur when machines with similar oscillations are mounted on the same base or when rotating shafts with close speeds share the same bearings. In these mechanical systems, forces interference from each source of vibration occur and a conventional data acquisition process for vibration analysis and dynamic balancing does not present satisfactory results. In this paper we present a simplified procedure to perform vibration analysis and dynamic balancing in systems subjected to beats frequency. This procedure is based on adjusting the data acquisition configuration parameters using conventional hardware and proprietary software. The results obtained show that the procedure presented in this paper has good efficiency. In addition, it is possible to notice significant benefits in terms of cost, simplicity and speed, especially when comparing with other types of more complex approaches that are being used today to treat similar problems.

1. Introduction

In rotating machines mounted close to each other, which share the same base, or, in mechanical systems that have rotation shafts that share the same bearings, it is common for vibrations to suffer interference [1]. If the rotations are significantly different on the rotors, different frequencies are generated and a standard vibration analysis procedure can separate the frequencies and indicate the existing amplitudes without difficulty. However, when the vibration sources present frequencies close to each other, there is considerable difficulty in performing vibration analysis and dynamic balancing on the system due to the appearance of beats frequency. In these systems, forces interference from each source of vibration occur and the performance of conventional vibration analysis and dynamic balancing procedures do not present satisfactory results, since, when the unbalances from each source emerge, two or more vectors with small differences in frequency are produced simultaneously, making it difficult to effectively identify each one. As examples of applications that may be subject to beats frequency, we highlight centrifugal separators and aeronautical gas turbines with dual-rotors. Centrifugal separators feature two rotors that share the same bearings and are widely used in many areas, such as in the chemical/pharmaceutical industry and in environmental protection systems. They enable continuous large-scale operation with a high efficiency factor in the separation of substances, particularly for solid–liquid mixtures. The vibration characteristics in centrifugal separators are governed by the coupled effect of the vibration energies of each rotor, being somewhat similar to the mechanical systems that make up aeronautical gas turbines with dual-rotors [2].

Statistics show that more than 60% of failures in modern rotating machinery are caused by misalignments in the rotating system [3]. Misalignments in rotating system are caused mainly by manufacturing and/or assembly problems, as well as by gaps caused by unbalance. Characteristics of the bearings significantly influence the vibratory behavior of the mechanical system under consideration [4,5]. In addition, structural integrity analyzes using computer simulations on the main mechanical components are important to avoid the occurrence of other types of failures [6,7]. In this paper, we address the problem of mass unbalance in rotating machines that can be mitigated by vibration analysis and dynamic balancing. Considering the arrangement of dual-rotors, it is known that its use in practice is based on the principle of maximizing efficiency by the use of more compact dimensions. In this context, Guskov et al. [8] investigated the vibration characteristics of structures with dual-rotors by means of numerical simulations and experimental tests. Ferraris et al. [9] analyzed the dynamic behavior of

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an asymmetric double rotor system using Campbell diagrams and responses to unbalances. Childs [10] studied the transient response of mechanical systems with dual-rotors considering the interaction of the effects of each source of vibration on the resulting global behavior. For this work, we consider the existence of beats frequency due to the condition that each of the rotors or rotating machines works with very close speeds. Thus, we present a simplified procedure for vibration analysis and dynamic balancing that is based on the adjustment of signal acquisition parameters using conventional hardware and proprietary software. Thus, beats frequency can be captured accurately and the vibration signals from each source are separated and analyzed in order to improve the overall behavior of the system. The results obtained and presented in this paper show that the proposed procedure has very good efficiency. In addition, it is possible to notice significant benefits in terms of cost, simplicity and speed. Especially when comparing the proposal presented in this paper with other types of more complex approaches that are currently being used to treat similar problems, such as the single point discrete Fourier transform that is based on cross-correlation theory [11] and the separation method based on the adoption of relative coordinates [12].

2. Related practical applications

Figs. 1 and 2 show schematic illustrations of a centrifugal separator system and a dual-rotor aeronautical turbine, respectively. For the centrifugal separator, the outer cylinder rotates at high speed to generate the centrifugal effect. Internally, a spiral rotates at a slightly greater rotation, pushing the solid parts towards the outlet. The speeds used for these rotors depend on the type of solid–liquid mixture being filtered, but the difference in speed of rotation between these two rotors is often very small. For the dual-rotor aeronautical turbine system, the internal rotor is considered a low pressure rotor and the external rotor is considered a high pressure rotor. In some specific situations, the difference in speed of rotation between these two rotors can also be very small.

Each of the rotors shown in Figs. 1 and 2 presents an independent unbalance and, even if they are within acceptable limits after manufacturing, when they work together, the sum of the existing unbalances can generate above ideal global levels of vibration, right after assembly or after a period of use due to wear. In experimental terms, tachometer and accelerometer sensors are used to capture the vibration signal at points of the mechanical system of greatest interest, usually on the bearing support structures. Subsequently, these acquired signals are traced with appropriate computational tools in order to obtain useful information for the design, maintenance or improvement of these systems. In the existence of beats frequency, the amplitudes of vibration add up and subtract each one depending on the moment of each force vector. When using a conventional vibration meter, the overall vibration value is indicated at each moment and, thus, the values fluctuate without a representative average that can be used for analysis. Therefore, in many practical situations it is not possible to perform a satisfactory vibration analysis and dynamic balancing using a conventional approach with conventional hardware and software.

3. Background mathematical modeling

3.1. Beats

When two harmonic motions, with frequencies close to one another, are added, the resulting motion, as already mentioned, exhibits beats. For example, if

$$x_1(t) = X\cos\omega t \tag{1}$$

$$x_2(t) = X\cos(\omega+\delta)t \tag{2}$$

where δ is a small quantity, the addition of these motions yields

$$x(t) = x_1(t) + x_2(t) = X[\cos\omega t + \cos(\omega+\delta)t] \tag{3}$$

using the relation

$$\cos(a) + \cos(b) = 2\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right) \tag{4}$$

Eq. (3) can be rewritten as

$$x(t) = 2X\cos\frac{\delta t}{2}\cos\left(\omega + \frac{\delta}{2}\right)t \tag{5}$$

This equation is shown graphically in Fig. 3. It can be seen that the resulting motion, $x(t)$, represents a cosine wave with frequency $\omega+\delta/2$, which is approximately equal to ω and with a varying amplitude of $2X\cos(\delta t/2)$. Whenever the amplitude reaches a maximum, it is called a beat. The frequency (δ) at which the amplitude builds up and dies down between 0 and $2X$ is known as beat frequency.

3.2. Fast Fourier Transform

The Discrete Fourier Transform is obtained by decomposing a sequence of values into components of different frequencies. Calculating this operation directly from the definition is often too slow to be practical. A Fast Fourier Transform is a way to calculate the same result more quickly: calculating the Discrete Fourier Transform of N points in the direct way, using the definition, takes arithmetic operations of $O(N^2)$, while a Fast Fourier Transform can compute the same Discrete Fourier Transform in just $O(N\log N)$ operations. It is possible to express the Discrete Fourier Transform as being

$$F(\omega) = \sum_{t=0}^{N-1} f(t) W_N^{\omega t} \tag{6}$$

where

$$W_N^{\omega t} = e^{-i2\pi\omega t/N} \tag{7}$$

For the Fast Fourier Transform, we assume that $N = 2^n$ where n is a positive integer. Therefore, N can be written as $N = 2M$ where M is also a

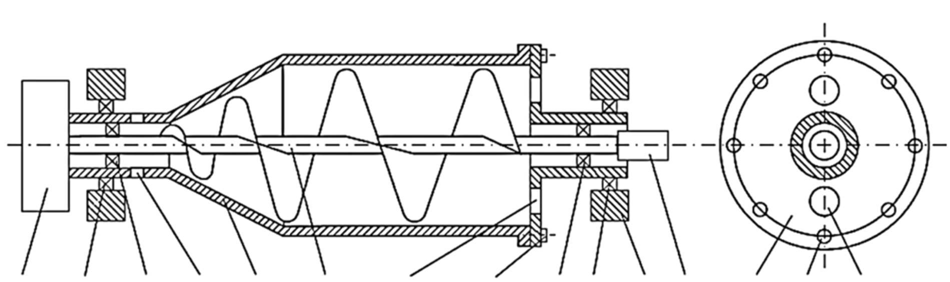

Fig. 1. Sketch of a spiral centrifuge adapted from [11]. 1. Gear box reducer; 2. Ball bearing; 3. Ball bearing; 4. Solid outlet; 5. Outer drum rotor; 6. Inner spiral rotor; 7. Liquid outlet; 8. Fixing bolt; 9. Ball bearing; 10. Ball bearing; 11. Bearing housing; 12. Extension part of inner rotor; 13. Right end of drum.

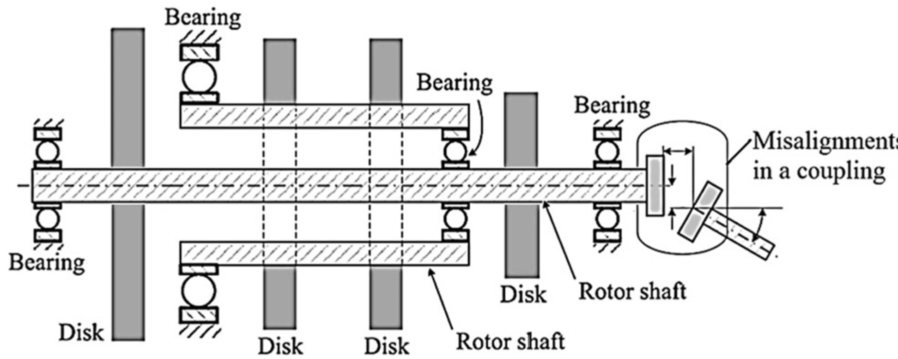

Fig. 2. Sketch of a dual-rotor aeroengine system adapted from [1].

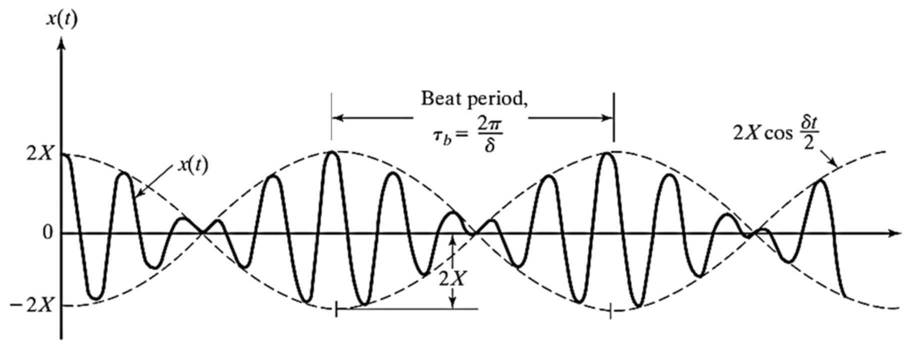

Fig. 3. Phenomenon of beats.

positive integer. Therefore, the Discrete Fourier transform initially written, can be rewritten as

$$F(\omega) = \sum_{t=0}^{2M-1} f(t) W_{2M}^{\omega t} \tag{8}$$

the sum written above can be separated into two, as follows

$$F(\omega) = \sum_{t=0}^{M-1} f(2t) W_{2M}^{\omega(2t)} + \sum_{t=0}^{M-1} f(2t+1) W_{2M}^{\omega(2t+1)} \tag{9}$$

We name the first sum by

$$F_{even}(\omega) = \sum_{t=0}^{M-1} f(2t) e^{-i2\pi\omega t/2M} \text{for} \omega = 0, 1, 2, \cdots, (M-1) \tag{10}$$

and the second sum by

$$F_{odd}(\omega) = \sum_{t=0}^{M-1} f(2t+1) e^{-i2\pi\omega(2t+1)/2M} \text{for} \omega = 0, 1, 2, \cdots, (M-1) \tag{11}$$

We can then rewrite the Fast Fourier Transform as being

$$F(\omega) = F_{even}(\omega) + F_{odd}(\omega) \tag{12}$$

Observing these equations gives us their properties. Among them we see that a transformation of N points can be computed by dividing the original expression into two parts.

3.3. Mass balancing

The mass balancing process should be done in a rotating mechanical system when the geometric center and the center of mass of the system do not coincide, causing meaningful disturbances. It is possible to calculate the force vector originated by this unbalanced mass using the equation

$$F = mRw^2 \tag{13}$$

where m is the unbalanced mass, R is the radius from the geometric center to the unbalanced mass and w is the angular speed of the rotating element. Using an accelerometer sensor and a tachometer sensor with a reflective tape it is possible to acquire the force vector raised by the unbalanced mass: the level of vibration in RMS represents the amplitude of this vector and the marking given by the reflective tape provide an indication of this vector phase. The RMS vibration level of a signal can be obtained by

$$\text{RMS} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} {v_i}^2} \tag{14}$$

where v_i are the instantaneous vector amplitudes of the considered waveform and N is the total number of points to be considered. As in the initial stage there is not an exact reference for the phase value of the signal (the position of the reflective tape and the position of the tachometer sensor are not perfectly synchronized with the initial reading of the vibration levels) it should be added a test mass in any known position of the rotating element to evaluate the disturbance that it causes in the system. The vectors can be measured: $V0$, $(V0 = X0@P0)$ for unbalanced mass only; and, VR, $(VR = XR@PR)$ for unbalanced mass plus test mass. Both $V0$ and VR can be measured using the RMS amplitude calculation. The $P0$ and PR phase angles can be indicated by the reflective tape and the tachometer sensor. VMT, $(VMT = VR - V0)$, can be calculated, which is a vector that exclusively represents the force generated by the test mass. The test mass, then, can be removed. The angular displacement from the initial position of the test mass and the value of the correction mass for balancing can be determined considering a correction vector VC, $(VC = XC@PC)$, such that, $VC + V0 = 0$. The correction mass value can be calculated by the product of the test mass with $XC/X0$. This cycle should be repeated until the appropriate level of balance is reached for the application.

4. Experimental analysis

In order to study the beats frequency problem, we mounted two 1/4 HP power motors on the same base, both controlled by two frequency inverters, Fig. 4. Unbalanced discs were coupled to these motors in order to generate two distinct sources of vibrations, conceptually simulating practical applications such as the centrifugal separator and the dual-rotor aeronautical gas turbine. Additionally, a NK820 Teknikao analyzer and a SDAV Teknikao proprietary software were used, in conjunction with a tachometer optical sensor with accuracy less than 0.1% and a piezoelectric accelerometer sensor with sensitivity of 100 mV/g which can be used at frequencies of 0.8 Hz to 10 kHz. Regarding the uncertainty of our experiments, we can state that the uncertainty of our experiments is below 2% according to Intermetro Calibration Laboratory from CGCRE in accordance with ABNT NBR ISO/IEC 17025.

4.1. Initial measurements

Initially, global measurements of vibration in mm/s (RMS) were performed. This speed unit has the characteristic of being better associated with the vibration energy that is given as a function of centrifugal force and mobility. Fig. 5 shows several measurements of the global levels of vibration in some situations of interest as a function of time. The piezoelectric accelerometer sensor is installed on the horizontal direction of the bearing of motor 2 near the unbalanced disk, according to Fig. 4. Thus, seeing Fig. 5, it is possible to clearly understand the beats frequency problem in vibration analysis and, consequently, in the dynamic balancing of mechanical systems like the ones addressed in this paper.

When only one of the motors is running and the other one is off, whether motor 1 or motor 2 is running, it is possible to measure globally the levels of vibration as a function of time. When the two motors are running simultaneously close to each other in speed and sharing the same base, there is instability in the measurement of global vibration levels. This is because conventional global meters acquire the vibration signals over a period of time and process the data to calculate the value in RMS. However, when the effect of one source interferes with the other, the overall level of vibration is different at each instant, since the force vectors of each source is in different relative positions. Still, vibration sources are known to have differences and asynchronicity. At each moment, the resulting vector changes in amplitude and in angle according to the possibilities of combination presented in Fig. 6. This phenomenon has the characteristic of modulating the amplitude of the vibration.

In Fig. 7(a) it is possible to see a modulation on the amplitudes of the acquired vibration signal as a function of time using the experimental apparatus of this work, and for this interval considered, the overall level of vibration is 2.80 mm/s (RMS). Otherwise, for the interval considered in Fig. 7(b), the value calculated for the global vibration level is 3.13 mm/s (RMS). Due to beats frequency, for different moments of acquisition different global levels of vibration are obtained.

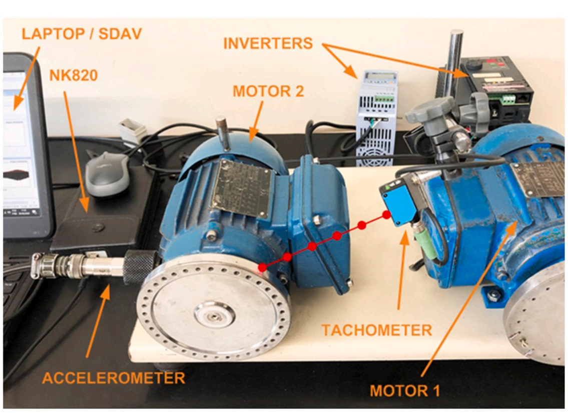

Fig. 4. Experimental apparatus used to study the frequency beat problem when two rotors have very close rotation speeds.

4.2. Acquisition of signals with frequency beat

Considering the two motors turned on, the data acquisition parameters are adjusted using proprietary software to perform vibration analysis and dynamic balancing on the studied system. With these adjustments in the acquisition parameters, it is possible to separate the signals from each source and to treat the problem using conventional hardware. Conventional vibration analysis and dynamic balancing hardware have a time of reading of about 1 s (up to 2 s in some equipment), which is insufficient to adequately capture beats frequency in most cases. To observe the beats frequency phenomenon, it is necessary to observe the signals for a longer period of time. This can be done by increasing the number of samples, increasing the time between acquisitions, or both. The beats frequency has a repetition cycle that is given by the difference between the frequencies of the vibration sources, as stated in Section 3.1. In the experiment presented, as motor 1 rotates at 3000 rpm and motor 2 rotates at 2960 rpm, the beats frequency phenomenon occurs at 40 rpm, that is, every 1.5 s. In acquisitions with a shorter period of time, it is not possible to analyze the phenomenon. In time, we recommend that the acquisition time be at least 3 times that given by the difference between the frequencies of the sources. Therefore, in the system under study, the acquisition must last at least 4.5 s. In Fig. 8, it is possible to observe the signal in the frequency domain with an acquisition period shorter than 1 frequency beat cycle. The difference between 3000 rpm and 2960 rpm can be seen, but without sufficient resolution for a good analysis. Typically, instruments for field balancing are manufactured with a 3 dB bandpass filter at 7% in relation to the rotation frequency of the machine to be considered. That is, this filter defines a margin of 7% above and below the main frequency to be analyzed. Thus, there must be an attenuation of 3 dB in the acquired signal to frequency values 7% above or below the main frequency considered. So, as can be seen, this specification is not enough to capture the beats frequency phenomenon when the source frequencies are very close. Furthermore, in the case of conventional acquisition hardware, it is not possible to acquire signals with a time long enough to analyze the beats frequency phenomenon. These instruments have a constant acquisition rate that leads to an increased number of acquisitions far above the usual available memory capacity of these devices.

To acquire the signal, we use a 12-bit Digital Analog Converter with a maximum frequency of 100 k samples/second. The microcontroller performs one conversion at a time. The number of points and the interval between conversions must be defined by the user according to the frequency of the alternating signal that is to be acquired; and this conversion must be at least twice as fast as the signal to be converted. In our case, we chose a conversion rate of 2.5 times the maximum frequency to be acquired. Thus, a signal at the maximum frequency will have at least 2 points acquired. Signals with lower frequencies will be composed of a number of points inversely proportional to their frequency. In this way, we obtain an excellent signal resolution for an accurate analysis, mainly at the operating rotating frequency which is the frequency necessary for performing the mass balancing. For example, to acquire a signal at the rotation of 3000 RPM (50 Hz) we need to define a maximum frequency, F_{max}, above this value, so that do not exist acquisition error (aliasing). The acquisition time T_{aq} for each point is then defined by $T_{aq} = 1/(F_{max} \times 2.5)$. For a maximum frequency of 500 Hz, $T_{aq} = 0.8\text{ms}$. It is now possible to define the number of points so that the total signal acquisition time be significantly longer considering the repetition of the beats frequency phenomenon. This time, T_{total}, is defined by $T_{total} = T_{aq} \times N$; where N is the number of points considered in the signal. The NK820/SDAV acquisition systems used have options to choose the number of

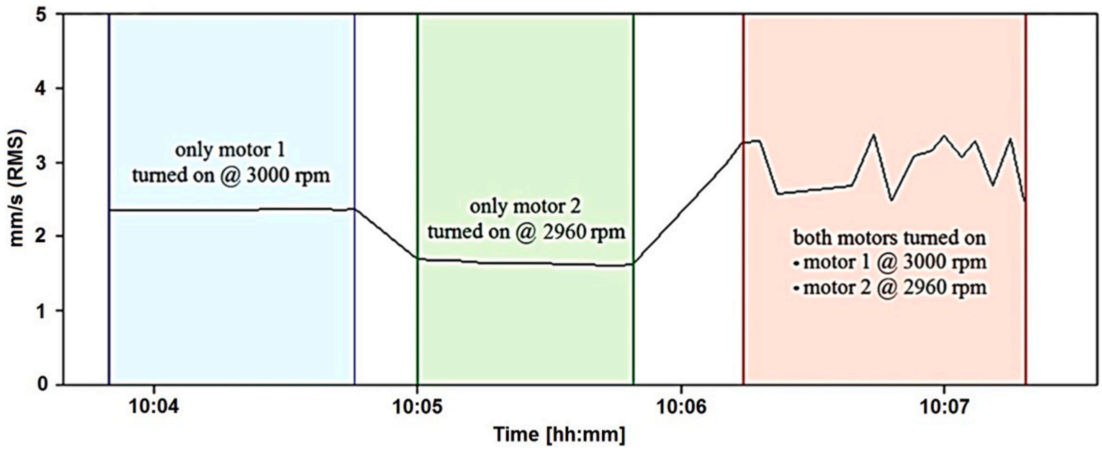

Fig. 5. Measurements of global levels of vibration as a function of time considering some situations of interest and the experimental apparatus developed in this work.

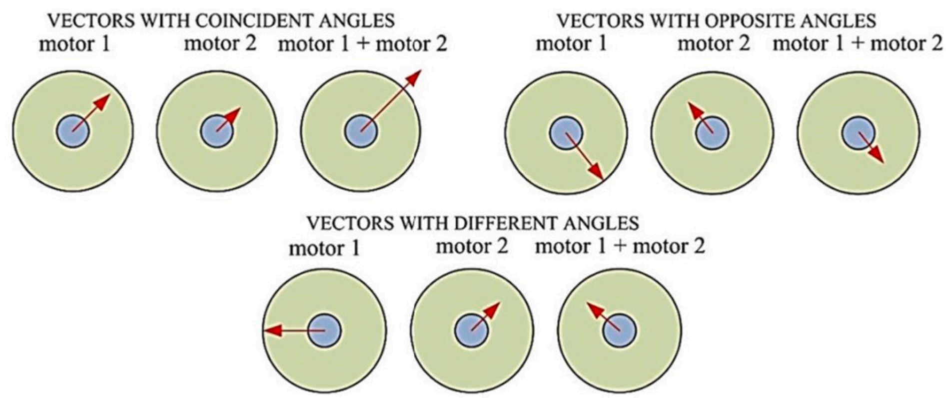

Fig. 6. Possibilities of combining force vectors from two different sources.

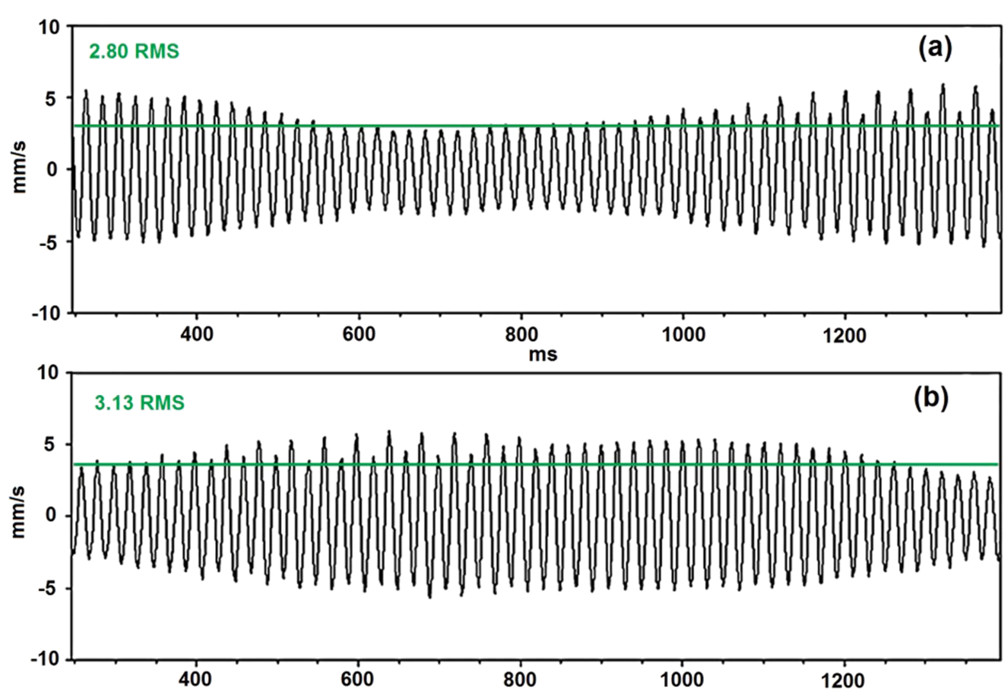

Fig. 7. Measurement intervals of vibration signal as a function of time to determine the overall level of vibration of the system. (a) Overall vibration level is 2.80 mm/s (RMS). (b) Overall vibration level is 3.13 mm/s (RMS).

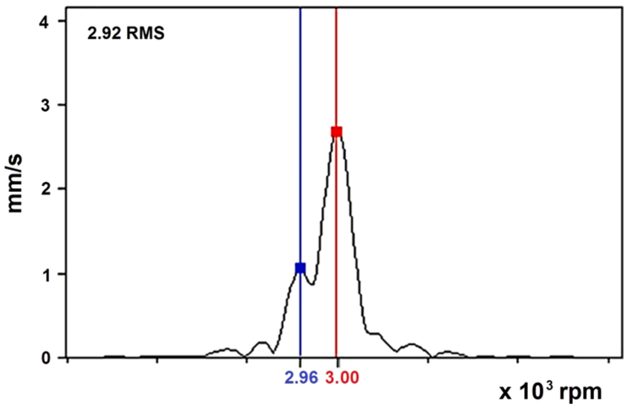

Fig. 8. Signal in the frequency domain for an acquisition period of 1.6 s.

points at 1 k, 2 k, 4 k, 8 k, 16 k and 32 K. For the maximum frequency to be acquired, the acquisition system has the following options: 50 Hz, 100 Hz, 200 Hz, 500 Hz, 1 kHz, 2 kHz, 5 kHz and 10 kHz.

When we set the number of points to be acquired to 2 k and the maximum frequency at 500 Hz, which theoretically would be much more than enough for the application, we have a total acquisition time of 1.6 s. However, this acquisition time is insufficient to capture beats frequency properly and to provide the necessary resolution to separate the signals, as shown in Fig. 8. That is, the analysis of the problem with the beats frequency cannot be performed. To get around the situation, we adjusted the acquisition parameters by choosing 16 k samples with a maximum frequency of 500 Hz, which provides a total acquisition time of 12.8 s. Thus, the beats frequency is well captured and the recommendation to collect at least 3 repetition cycles of the phenomenon is respected. The 500 Hz scale (maximum frequency) was chosen so that the acquired signal is composed of many points, providing a high resolution, once this way, after the Fourier transform, the result is clearer. In Fig. 9 it is possible to see the signal acquired and presented in the time domain. This signal represents the sum of the vibration effects of the two motors. The cursors are positioned in a frequency beat cycle and show a difference of 40 rpm. With the adjustments made into the acquisition parameters, specifically the definition of the number of points to be acquired and the time of acquisition, it is possible to capture a signal with much more quality and, thus, separate the frequencies of interest very accurately, Fig. 10.

4.3. Digital filter of frequencies

Considering the acquired signal presented in Fig. 10 from the adjustments in the data acquisition parameters, a vector was obtained for the stable signal with constant amplitude and phase. This allows performing a standard vibration analysis and dynamic balancing by separating the signals of interest from each source as a function of time, even considering the two motors connected simultaneously and knowing that one source of vibration generates interference in the other, which affects

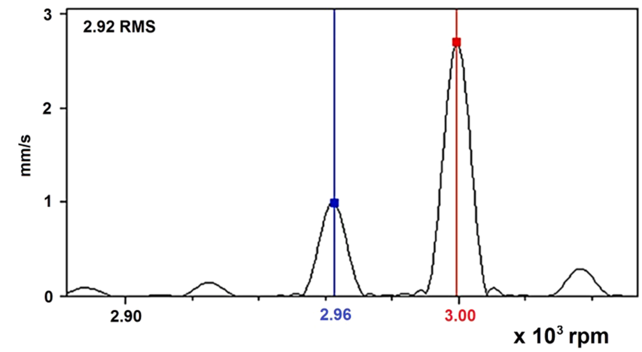

Fig. 10. Signal in the frequency domain for an acquisition period of 12.8 s.

the global behavior. In Fig. 11 it is possible to see the signal as a function of frequency with the cursors defining the frequency bands to be filtered. In the proprietary software SDAV there is a digital frequency filter that allows separating the frequencies as can be seen in Figs. 12–14.

The tachometer optical sensor with reflective tape and the piezoelectric accelerometer sensor were installed to work on the bearing and the disk of motor 2, see Fig. 4. Fig. 12 shows the filtered signal in the time domain from the delimitation of 2938–3030 rpm, which refers to the interaction of the effects of the vibration sources of the two unbalanced motors. It can be clearly observed the amplitude modulation of signal and the detection of reflexive tape by the tachometer optical sensor as detached by the markers shown in red. Fig. 13 shows the filtered signal in the time domain from the delimitation of 2989–3013 rpm, which refers to the source of vibration of motor 1. As the disk in motor 1 do not have tachometer optical sensor with the reflective tape, constant amplitude and variable phase can be observed. In this way, the average vector tends to zero. Fig. 14 shows the filtered signal in the time domain from the delimitation from 2947 to 2973 rpm, which refers to the vibration source of the motor 2. In this case, for motor 2, the tachometer optical sensor with the reflective tape was installed. So, it can be observed constant amplitude and phase, see markers in red. In this way, it is possible to use the resulting vector for the dynamic balancing procedure. Dynamic balancing will take a little bit longer due to the slower acquisition, and it does not depend on faster data processing, or computational capacity. Therefore, it is necessary to wait for the time of the beats frequency phenomenon to occur.

4.4. Dynamic balancing procedure

In the pulses intervals detected by the tachometer optical sensor there is a complete revolution equivalent to 360°. Using the Fourier transform, it is possible to accurately determine the amplitude of the sinusoid signal in relation to the vibration level and its respective phase angle relative to the passage through the reflective tape. The amplitude will be proportional to the mass that causes the unbalance and the angle will be proportional to the position of that mass. The amplitude and angle values can be represented by a vector, named zero vector ($V0$).

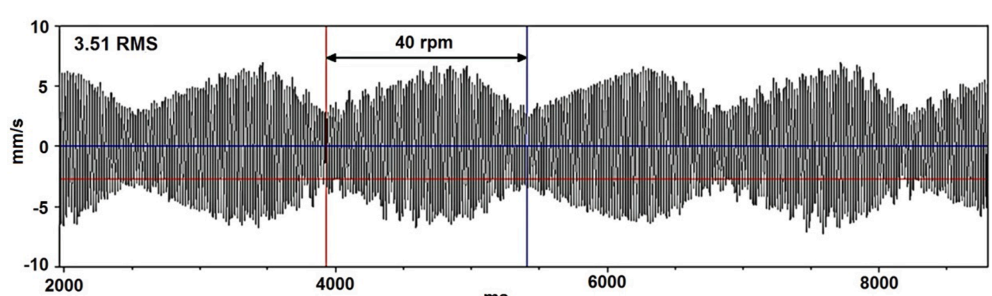

Fig. 9. Signal in the time domain representing the beats frequency phenomenon.

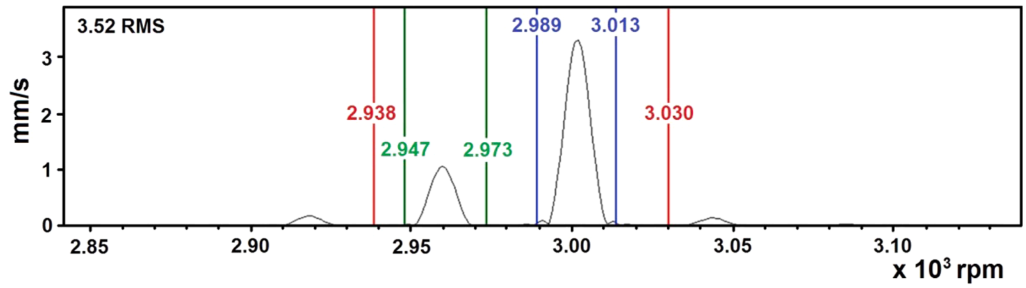

Fig. 11. Signal in the frequency domain collected using adjusted acquisition parameters with delimitations of the frequency bands to be filtered.

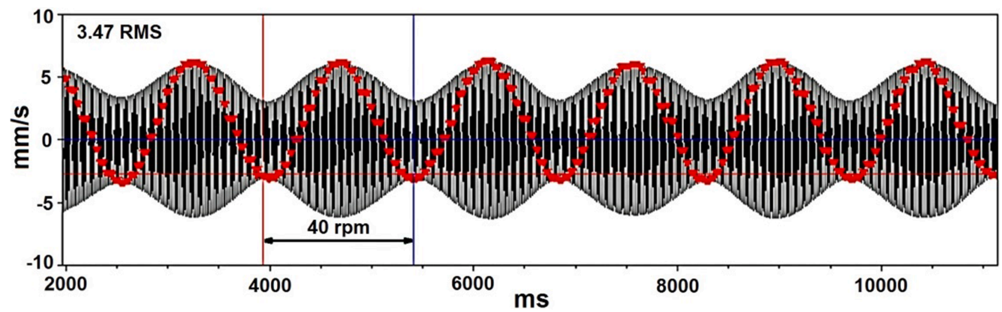

Fig. 12. Filtered signal in the time domain regarding the interaction of the effects of the vibration sources of the two unbalanced motors.

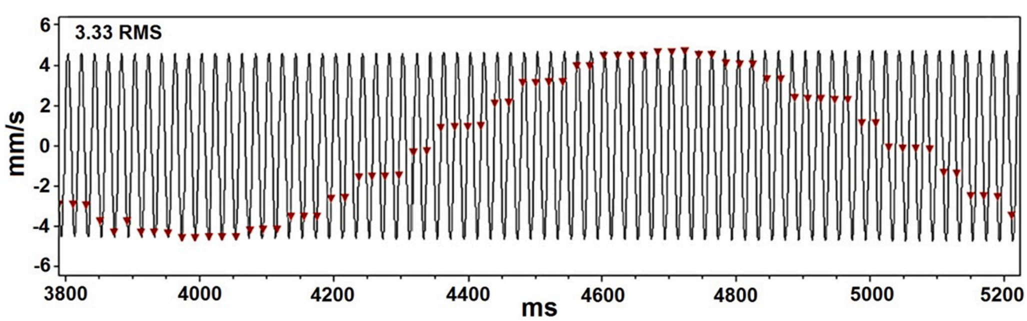

Fig. 13. Filtered signal in the time domain referring to the vibration source of motor 1.

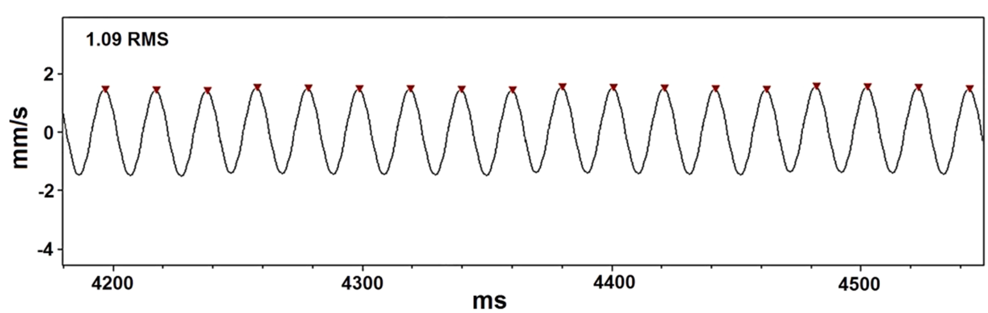

Fig. 14. Filtered signal in the time domain referring to the vibration source of motor 2.

Using the developed SDAV software, it is possible to obtain the amplitude and phase of the filtered signal as shown in Fig. 14. For the amplitude, the value is 1.09 mm/s (RMS) and for the phase, the value is 190° ($V0 = 1.09@190^{\circ}$). To start the balancing process, a known mass is added in a position also known without relation to the origin established for $V0$. This added test mass will combine with the unbalance mass, generating a new vector, named resulting vector (VR). In the case, a test mass of 0.61 g was added using a radius of 65 mm onto the disk of motor 2. The quality balancing grade required for the application in study is determined according to ISO1940 G-2.50. With the increase of the test mass of 0.61 g, the resulting vector was calculated and presents an amplitude of 1.21 mm/s (RMS) with a phase of 264° in relation to the origin used to represent $V0$ ($VR = 1.21@264^{\circ}$), as can be seen in Fig. 15 (a). By subtracting the two vectors ($VR - V0$), it is possible to determine the vector generated by the test mass alone (VMT) and to establish the location of the new origin of reference (0°). It is possible to observe that this VMT vector was not generated at the opposite angle to $V0$, therefore, the test mass must be moved by an angle θ, as shown in Fig. 15(b).

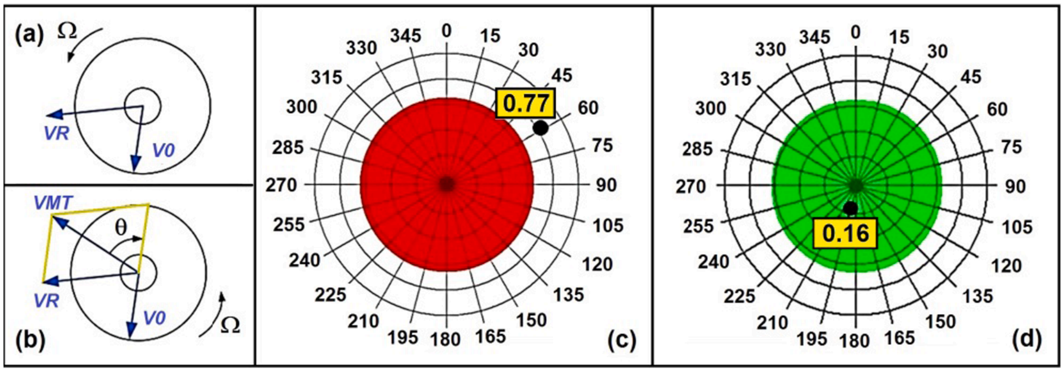

Fig. 15. Dynamic balancing process on motor 2: (a) Schematic representation of ***V*0** and ***VR***; (b) Schematic representation of obtaining ***VMT*** and angle θ; (c) First correction mass of 0.77 g at an angle of 59° contrary to the direction of disk rotation; (d) Vibration levels in accordance with ISO1940 G-2.5.

The correct amount for the first unbalance correction mass is given by the relationship between the amplitudes of the *VMT* and *V*0 vectors multiplied by the test mass. Then, the software indicates for the unbalance correction mass the value of 0.77 g at an angle $\theta = 59^{\circ}$ in the opposite direction to the rotation of the disk (Ω), as shown in Fig. 15(b) and 15(c). After removing the test mass and adding the correction mass of 0.77 g at the specified position, vibration levels in the order of 0.22 mm/s (RMS) were obtained, i.e., a reduction of approximately 82% in the level of vibration, complying with ISO1940 G-2.5 as can be seen in Fig. 15(d). This process can be repeated as many times as necessary until the desired vibration level is reached. For exemplification purposes, if other iteration was made to further reduce the level of vibration, the correction mass of 0.77 g should be maintained and an additional correction mass of 0.16 g @ 195° should be added from the new origin defined by the correction mass of 0.77 g. This would further reduce the vibration level of motor 2.

5. Results

After performing the dynamic balancing on the disk of motor 2, a significant attenuation of the influence of this source of vibration in relation to the global signal of the system can be perceived, when the two motors are running simultaneously, see Fig. 16(a). In a complementary way, Fig. 16(b) shows the global signal of the system after performing the dynamic balancing on the disk of motor 1. This attenuation of the vibration levels by means of dynamic balancing on motor 1 has been performed putting the reflective reference tape of the tachometer optical sensor on the disk of motor 1, since the phase reference must remain on the motor to be balanced. Thus, a significant attenuation in the source of vibration relative to the motor 1 can be observed. Fig. 17 shows the overall system signal acquired in the time domain for an interval of 32 s. It can be seen that there was a severe reduction in the overall level of vibration of the system under study.

6. Conclusions

In this paper, we present a simplified procedure based on the use of conventional hardware with adjustments in the data acquisition parameters to accurately capture and treat beats frequency phenomenon. To observe the beats frequency phenomenon, it is necessary to observe the signals for a longer period of time. The beats frequency has a repetition cycle that is given by the difference between the frequencies of the vibration sources. To adequately capture beats frequency, we recommend adjusting the acquisition parameters to reflect an acquisition period of at least 3 times the difference of the vibration frequencies sources. This can be done by increasing the number of samples, increasing the time between acquisitions, or both. Once the signal representing the problem is properly acquired, the main sources of vibration can then be separated and analyzed in order to characterize the influence of each one on the overall vibratory behavior of the system. The values of the magnitudes and phases of the unbalances can be obtained by sampling the vibration signals, which allows a subsequent dynamic balancing of each source, even when these sources are operating simultaneously. The results obtained with this work show that by using the strategy presented in this paper, a considerable advantage can be perceived in terms of cost, simplicity and speed, mainly, when comparing the presented proposal with other types of more complex approaches that are currently being used.

CRediT authorship contribution statement

Luciano Paiva Ponci: Investigation, Formal analysis, Resources. **Geraldo Creci:** Formal analysis, Resources. **João Carlos Menezes:** Resources, Supervision.

Declaration of Competing Interest

The authors declare that they have no known competing financial

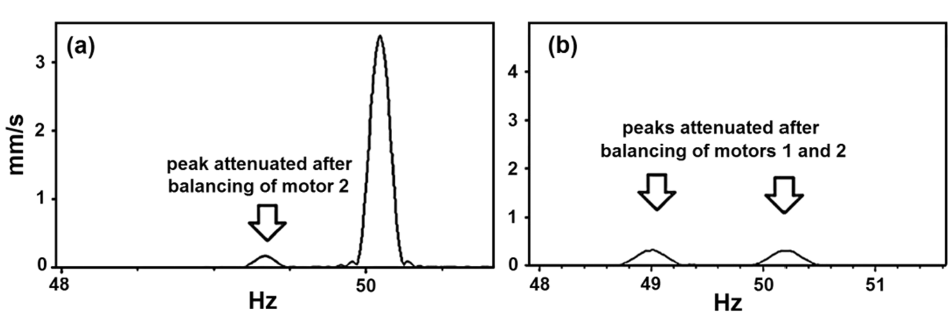

Fig. 16. Signals acquired in the frequency domain representing the two main sources of vibration (a) Highlight for the attenuation in the peak relative to motor 2 after balancing of the disk of motor 2 only; (b) Highlight for the attenuation of the two peaks after the balancing of the two disks of the motors.

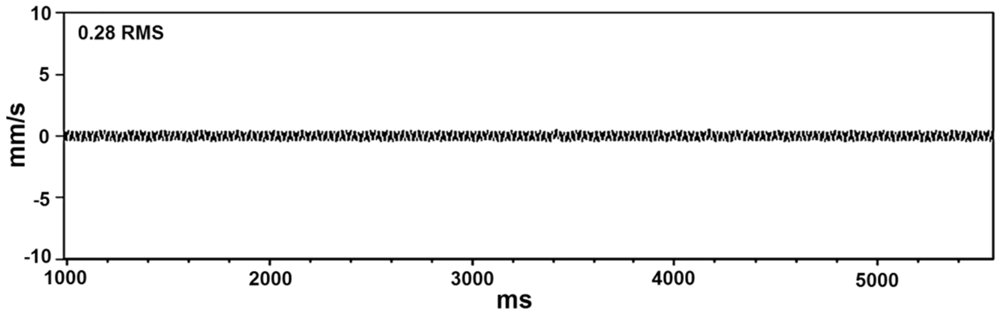

Fig. 17. Global signal of the system acquired in the time domain for an interval of 32 s after the balancing of the disks of motors 1 and 2.

interests or personal relationships that could have appeared to influence the work reported in this paper.

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